Two-layer Sliding Mode Control of Pneumatic Position Synchro System with **Feedback Linearization Based on Friction Compensation**

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Abstract – Pneumatic systems have been widely used in industrial applications because of their well-known advantages. However, pneumatic systems have disadvantages that include strong nonlinearity and low natural frequency. These drawbacks make it difficult to obtain satisfactory control performances in comparison to hydraulic systems. In this paper, the fundamental characteristics and nonlinear control strategy of the pneumatic system is studied. The two-layer sliding mode synchro system with feedback linearization based on friction compensation is applied to electropneumatic synchro position system. To verify the used strategies, experiments with the synchro system are performed. The experimental results show that the new designed controller is effective for both accuracy and robustness.

Keywords – Pneumatic servo system, Feedback linearization, Sliding mode control.

I. INTRODUCTION

With the development of aerospace technologies and modern machine manufacturing, the pneumatic servo control system is widely used in numerous engineering areas. Pneumatic actuators also have the advantage of clean operation and easy serviceability. In most cases, the application is limited to point to point control. The disadvantages of the pneumatic systems mainly include air compression, strong non-linearity, and large mechanic friction. These drawbacks restrict their applications in servocontrol systems. However, new requirements in pneumatic system specifications are continually being proposed. Recently, there have been many efforts that were directed towards meeting these requirements. Some researchers focus on pneumatic system model study. Scavanda [1] and Liu [2] derived a nonlinear model for the pneumatic system with relevant experiments to test the model. Lu [3] developed an accurate mathematical model of the servo-system through analysis of the flow characteristics of the proportional flow valve and the friction of the cylinder. Due to the high friction in the pneumatic system, the disturbances led to uncertainties in the system. This problem is considered as the main difficulty in pneumatic control. Sliding mode control has been used as robust nonlinear control technique in many applications involving nonlinearity uncertainty in systems [4,5]. A better performance is achieved by using the discontinuous control rule with low gains. Precise position synchro systems are also required. As a matter of fact, hydraulic synchro position systems are thoroughly studied in references [6-9]. The hydraulic synchro system has several advantages, such as high precision, large load; but the system is complex and expensive. The major advantages of pneumatic synchro systems include the sufficiency of simple and low-cost equipment, and are also convenient for practical applications and maintenance. However, the synchro accuracy is not satisfactory and the load capacity is small.

In this paper, the electro-pneumatic position synchro servo control system is introduced, and the fundamental characteristics and nonlinear control strategy of the pneumatic system are studied. The two-fold sliding mode synchro controller with friction compensation is applied to the electro-pneumatic synchro position system. The effectiveness of the proposed technique is demonstrated by experimental implementation on a two-cylinder synchro system.

II. SYSTEM DESCRIPTION

A. The Configuration of the Electro-pneumatic Position Synchro Servo Control System

The servo control system consists of two rod-less pneumatic cylinders which are controlled with two proportional directional control valves as shown in Fig. 1(a) and (b). Feedback equipments include pressure sensors and two displacement encoders. Pneumatic supply conditioning equipments provide filtering and regulation. The user interface is provided via a terminal.

The components in the system are as follows:

- (1) Two Festo electro-pneumatic five-port proportional valves: MPYE-5-1/4-010B;
- (2) Two rod-less pneumatic cylinders with stroke length of 600mm: DGP-40-600-PPV-A-B;
- (3) One air source;
- (4) Two pneumatic accessories: LFR-D-5M-MIDI;
- (5) Four pressure sensors: PT351-0.6MPa-0.3;
- (6) Two position sensors: MLO-POT-600-TLF;

(7) Mass load A (5.75 kg) and mass load B (5.85kg) mounted on sliding parts which are connected with pistons of cylinder A and cylinder B.

External forces (17N and 34N) are applied to cylinder A using a wire and a weight through a pulley in order to simulate an external load disturbance.



Fig. 1(a) Scheme of the experimental system



Fig. 1(b) Photos of the experimental table

III. MATHEMATICAL MODEL

A nonlinear state–space model of the system is derived based on a single cylinder pneumatic model from reference [10] as follows:

(1) Subsystem 1 (with cylinder A):

$$\dot{x}_1 = \dot{x} = x_2 \tag{1}$$

$$\dot{x}_2 = \frac{A_p}{M_1}(p_{x1} - p_{x2}) - \frac{F_f}{M_1} - \frac{F}{M_1}$$
(2)

$$\dot{x}_{3} = \dot{p}_{x1} = \frac{\frac{kRC_{a}C_{0}}{\sqrt{R/T}} \left[A_{1s}(u) \cdot p_{s} \cdot f\left(\frac{p_{x1}}{p_{s}}\right) - A_{1e}(u) \cdot p_{x1} \cdot f\left(\frac{p_{e}}{p_{x1}}\right) \right] - kA_{p}p_{x1}\dot{x}}{V_{10} + \frac{1}{2}A_{p}L + A_{p}x}$$
(3)

$$\dot{x}_{4} = \dot{p}_{x2} = \frac{\frac{kRC_{d}C_{0}}{\sqrt{R/T}} \left[A_{2s}(u) \cdot p_{s} \cdot f\left(\frac{p_{x2}}{p_{s}}\right) - A_{2e}(u) \cdot p_{x2} \cdot f\left(\frac{p_{e}}{p_{x2}}\right) \right] + kA_{p}p_{x2}\dot{x}}{V_{20} + \frac{1}{2}A_{p}L - A_{p}x}$$
(4)

where x is the piston output displacement of cylinder A; p_{x1} and p_{x2} are the pressures in left and right chamber, respectively; F is the external force; F_{f1} is the friction force of Cylinder A; and M_1 is the mass load of cylinder A.

(2) Subsystem 2 (with cylinder B)

$$\dot{y}_1 = \dot{y} = y_2 \tag{5}$$

$$\dot{y}_2 = \frac{A_p}{M_2} (p_{y1} - p_{y2}) - \frac{F_{f2}}{M_2}$$
(6)

$$\dot{y}_{3} = \dot{p}_{y1} = \frac{\frac{kRC_{d}C_{0}}{\sqrt{R/T}} \left[A_{1s}(u) \cdot p_{s} \cdot f\left(\frac{p_{y1}}{p_{s}}\right) - A_{1e}(u) \cdot p_{1} \cdot f\left(\frac{p_{e}}{p_{y1}}\right) \right] - kA_{p}p_{y1}\dot{y}}{V_{10} + \frac{1}{2}A_{p}L + A_{p}y}$$
(7)

$$\dot{y}_{4} = \dot{p}_{y2} = \frac{\frac{kRC_{4}C_{0}}{\sqrt{R/T}} \left[A_{2s}(u) \cdot p_{s} \cdot f\left(\frac{p_{y2}}{p_{s}}\right) - A_{2e}(u) \cdot p_{y2} \cdot f\left(\frac{p_{e}}{p_{y2}}\right) \right] + kA_{p}P_{y2}\dot{y}}{V_{20} + \frac{1}{2}A_{p}L - A_{p}y}$$
(8)

where y is the piston displacement output of cylinder B, p_{y1} and p_{y2} are the pressures in the left and right chamber of cylinder B, respectively; F_{f2} is the friction force; and M_2 is the mass load of cylinder B.

$$f\left(\frac{p_d}{p_u}\right) = \begin{cases} \sqrt{\frac{2}{k-1} \cdot \left(\frac{k+1}{2}\right)^{k+1/k-1} \cdot \left[\left(\frac{p_d}{p_u}\right)^{2/k} - \left(\frac{p_d}{p_u}\right)^{k+1/k}\right]} & \text{if} \quad p_{atm}/p_u \le p_d/p_u \le C_r \\ 1 & \text{if} \quad C_r \le p_d/p_u < 1 \end{cases}$$

R — Pneumatic Constant J/kg · K;

$$C_0 = \sqrt{k \cdot \left(\frac{2}{k+1}\right)^{k+1/k-1}}$$

 C_d is the valve orifice discharge coefficient, Cd = 0.8;

Critical pressure ratio $C_r = 0.528$; Specific heat k = 1.4 A_p is the piston area; V_{10} is initial volumes of the chamber and is the volumes of the long hole. L is the piston stroke. A_{1s} , A_{2s} , A_{1e} and A_{2e} are the valve geometric orifice area function, they are the nonlinear function of u [11].

Due to the complexity of the equations, the following linear functions are used for approximation

$$A_{1s}(u) = \pi D u \tag{9}$$

$$A_{2s}(u) = \pi D\delta \tag{10}$$

$$A_{1e}(u) = \pi D\delta \tag{11}$$

$$A_{2e}(u) = \pi D u \tag{12}$$

The requirement of synchro control of the two cylinders is that the displacement output error between subsystem 1 and subsystem 2 is less than the permitted tolerance. Therefore, the displacement outputs of subsystem 1 and subsystem 2 are approximately same. This requirement can be expressed as follows:

$$\max |x(t) - y(t)| \le e \qquad t \in [a, b] \tag{13}$$

where, e – permitted position tolerance;

[a,b] – defined time region.

Since the two subsystems characteristics are not the same, the synchro control becomes complex. Therefore, the sliding mode control strategy with feedback linearization based on friction compensation is proposed. The control system design and the experimental setup and results are presented in the following section.

IV. DESIGN OF CONTROL STRATEGY AND EXPERIMENTAL RESULTS

For the past few years, nonlinear control theories play an important role in control engineering. The feedback linearization method developed rapidly, and there are substantial research results in this area. Compared to other control theories, this method has been proven to be effective for numerous practical problems [12-13].

From the mathematical model presented in the previous section, it is clear that the pressure functions in the pneumatic cylinder are nonlinear. Therefore, feedback linearization can be implemented for the pressure functions, while the pressure drops are taken as the important layer of the sliding mode. The two-layer sliding mode control strategy with feedback linearization based on friction compensation is proposed to overcome the friction disturbance in the cylinder. This method can also overcome the friction effect through discontinuity of the two layer sliding mode. The speed of the system response is enhanced by using pressure feedback linearization. This also ensures attenuation of the pressure fluctuations. Therefore, system tracking accuracy and speed are achieved. The control rules are designed as follows:

Control rule in cylinder A:

$$(p_{x1} - p_{x2})A_p = M_1 \ddot{x} + F_f + F \tag{14}$$

Integrated layer of the sliding surface is:

$$\sigma_{n1} = \Delta p_{r1} A - M_1 \ddot{y}_d + M_1 c_1 \dot{e}_1 - K_{f1} \dot{x} - F_{Kc1} \operatorname{sgn}(\dot{x}) + W \operatorname{sgn}(s_1) \quad (15)$$

The control rule is:

$$u_1 = -x_{V1} \operatorname{sgn} \sigma_{n1}, x_{V1} > 0 \tag{16}$$

where $e_1 = y_d - x$; M_1 is the mass load of cylinder A; K_{f1} is the coefficient of viscous friction; F_{kc1} is Coulomb friction in cylinder A; s_1 and σ_{n1} are the first layer of the sliding mode and synthetic sliding mode in cylinder A, respectively; Δp_{r1} is the pressure drop in cylinder A; and W is the control coefficient of the first sliding mode layer (W=300).

Control rule in cylinder B: First layer of the sliding surface is:

$$s_2 = \dot{e}_2 + c_2 e_2, \ c_2 > 0 \tag{17}$$

Integrated layer of the sliding surface is:

$$\sigma_{n2} = \Delta p_{r2} A - M_2 \ddot{y}_d + M_2 c_2 \dot{e}_2 - K_{f2} \dot{x} - F_{Kc2} \operatorname{sgn}(\dot{x}) + W \operatorname{sgn}(s_2) \quad (18)$$

The control rule is:

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$$u_2 = -x_{V2} \operatorname{sgn} \sigma_{n2}, x_{V2} > 0 \tag{19}$$

where $e_2 = y_d - x$; M_2 is the mass load of cylinder B; K_{f2} is the coefficient of viscous friction; F_{kc2} is Coulomb friction in cylinder B; s_2 and σ_{n2} are the first layer of sliding mode and synthetic sliding mode in cylinder A, respectively; Δp_{r2} is the pressure drop in cylinder B; and W is the control coefficient of the first sliding mode layer (W=300).

Furthermore, when large friction exists in the system, it is sometimes compensated using practical experience. The compensation rule that was used is:

$$x_{v} = \begin{cases} x_{vv} + \Delta_{1} & |e| > e_{d} \text{ and } x_{vv} > 0 \\ x_{v0} & |e| \le e_{d} \text{ or } x_{vv} = 0 \\ x_{vv} + \Delta_{2} & |e| > e_{d} \text{ and } x_{vv} < 0 \end{cases}$$
(20)

where, e_d - error tolerance;

 $x_{\nu\nu}$ - control value before the friction compensation; Δ_1, Δ_2 - friction compensation values.

Fig. 2 illustrates the experimental results including an ideal sinusoidal displacement input $[300+100\sin(0.314t)]$ [mm] and two displacement output responses x, y, which are responses of cylinder A and cylinder B, respectively. The system is controlled using classical PID controller with the following control parameters for cylinder A: $k_p = 6, k_i = 0.0015, k_d = 0.01$; and for cylinder B: $k_p = 7$, $k_i = 0.004, k_d = 0.015$.



Fig. 2 Tracking Synchro control with PID controller

It is clear from Fig. 2 that the output response curves are distorted and fail to track the sinusoidal input due to the effect of the non-linear friction forces. The experimental results show that the displacement synchro relative error between cylinder A and cylinder B is 15%.

Fig. 3 illustrates the experimental results of the displacement output responses of cylinders A and B performed with different displacement inputs. In this experiment, we used the two-layer sliding mode control strategy with feedback linearization based on friction compensation as proposed earlier. No external force is applied to cylinder A. Fig. 3 shows the step response (300mm step size), with maximum relative synchro error of 3%.

Fig. 4 shows the experimental results of sychro tracking sinusoidal displacement curve without external force. For the ideal displacement input [300+100sin(0.314t)] [mm], the maximum relative synchro error was 5%.

Fig. 5 shows the synchro tracking results with 17N external force on cylinder A, for which the maximum relative synchro error was 5%.



Fig. 3. The response of Synchro control without external force using proposed control strategy



Fig. 4. The tracking of the synchro control without external force using the new control strategy



new control strategy

According to the experimental results, the new control strategy using the two-layer sliding mode control based on feedback linearization with friction compensation performs much better compared to the PID controller. The synchro error was reduced and the control accuracy and speed were effectively enhanced.

V. CONCLUSIONS

The basic control characteristic of the synchro position system was studied and the synchro system model was derived in this paper. A two-layer sliding mode control on feedback linearization based on friction compensation was proposed. The experimental results show that the maximum relative synchro error of a step response without external force was 3%; and the maximum relative synchro error with external force was 6%. The maximum relative synchro error for tracking sinusoidal input without external force was 5%. The maximum relative synchro errors for tracking sinusoidal input without external force was 5% and 8%, respectively. It is evident that the proposed new control strategy has excellent integrated performance and can greatly reduce the synchro position error.

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